H. W. 3 SOLNS

SECTION 1.9

1. $x^2y^3 + x(1+y^2)\, dy/dx = 0$, $\mu = \frac{1}{xy^3}$

$\Rightarrow x + \left(\frac{1+y^2}{y^3}\right) dy/dx = 0$

$M_y = N_x = 0$ EXACT ✓

$\psi = \int M\, dx + h(y) = x^2/2 + h(y)$

$\psi_y = h'(y) = N = y^{-3} + y^{-1}$

$\Rightarrow h(y) = -\frac{1}{2}y^{-2} + \ln(y)$

$\therefore \psi(x,y) = \frac{x^2}{2} - \frac{1}{2}y^{-2} + \ln(y) = C$

(2) $y\,dx + (2x - ye^y)\,dy/dx = 0, \quad \mu = y$

$\Rightarrow \quad y^2 + (2xy - y^2e^y)\,y' = 0$

$M_y = 2y = N_x$ ✓ EXACT

$$\psi = \int y^2\,dx + h(y) = xy^2 + h(y) \Rightarrow \psi_y = 2xy + h'(y)$$

$$\psi_y = N = 2xy - e^y \cdot y^2 \Rightarrow h'(y) = -e^y \cdot y^2$$

$$\Rightarrow \quad h(y) = -y^2e^y + 2ye^y - 2e^y$$

$$\therefore \quad \psi(x,y) = xy^2 + (2y - y^2 - 2)e^y = C$$

(4) $\left(3x^2y + 2xy + y^3\right) + \left(x^2+y^2\right)y' = 0 \quad (*)$

$$M_y = 3x^2 + 3y^2 + 2x$$

$$M_y \neq N_x \quad \underline{\text{NOT EXACT}}$$

$$N_x = 2x$$

$$M_y - N_x = 3\left(x^2+y^2\right)$$

$$\Rightarrow \quad \frac{M_y - N_x}{N} = \frac{3\left(x^2+y^2\right)}{x^2+y^2} = 3$$

$$\mu = e^{\int (M_y - N_x)/N \, dx} = e^{\int 3\, dx} = e^{3x}$$

<u>NEW ODE</u> :

$$(*)\cdot \mu(x) = e^{3x}\left(3x^2y + 2xy + y^3\right) + e^{3x}\left(x^2+y^2\right)y' = 0$$

$$\psi = \int \left(3x^2y + 2xy + y^3\right)e^{3x}\, dx + h(y)$$

$$= e^{3x}\left(yx^2 + \frac{1}{3}y^3\right) + h(y)$$

$$\Rightarrow \psi_y = x^2e^{3x} + e^{3x}y^2 + h'(y) \quad \Rightarrow \quad h'(y) = 0 \quad \Rightarrow \quad h(y) = C$$

$$\therefore \quad \boxed{\psi(x,y) = e^{3x}\left(yx^2 + \tfrac{1}{3}y^3\right) = C}$$

SECTION 1.10

① $(t-3)y' + \ln(t)y = 2t, \quad y(1) = 2$

PUT IN FORM: $y' + p(t)y = g(t)$

TO GET:

$$y' + \frac{\ln(t)}{t-3}y = \frac{2t}{t-3}$$

$p(t)$ DISCONTINUOUS @ $t = 3$ AND FOR $t \le 0$.

$g(t)$ DISCONTINUOUS @ $t = 3$.

INITIAL CONDITION @ $t_0 = 1$

$\Rightarrow$ ~~SOL~~ UNIQUE SOL$^{\underline{n}}$ EXISTS ~~FOR~~ ON ~~THE~~ OPEN INTERVAL $(0,3)$

④ $y' = \dfrac{t-y}{2t+5y} \Rightarrow f(t,y) = \dfrac{(t-y)}{2t+5y}$

$$\frac{\partial f}{\partial y} = \frac{(-1)(2t+5y) - (t-y)(5)}{(2t+5y)^2}$$

$\Rightarrow f, \partial_y f$ DISCONTINUOUS ON LINE $y = -\frac{2}{5}t$.

$\therefore$ UNIQUE SOL$^{\underline{n}}$ EXISTS EVERYWHERE IN ty PLANE ~~EXCEPT~~ EXCEPT FOR ON THE LINE $y = -\frac{2}{5}t$

(8) $y' = 2ty^2$ $\quad \Rightarrow \quad y(t) = \dfrac{1}{-t^2+c} \quad \Rightarrow \quad y(t) = \dfrac{-1}{t^2 + 1/y_0}$

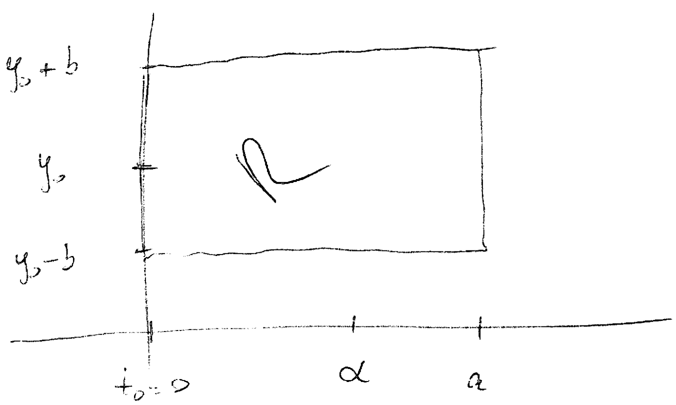
$y(0) = y_0$

$$\Rightarrow \quad \boxed{y(t) = \frac{-y_0}{y_0 t^2 + 1}}$$

$$\left.\begin{aligned} f(t,y) &= 2ty^2 \\ f_y &= 4ty \end{aligned}\right\} \text{ BOTH ARE CONTINUOUS IN ENTIRE } ty \text{ PLANE.}$$

FIX $a, b > 0$

$$M \equiv \max_{(t,y) \text{ IN } R} |f(t,y)| = \begin{cases} |2a(y_0+b)| & \text{IF } y_0 > 0 \\ |2a(y_0-b)| & \text{IF } y_0 < 0 \end{cases}$$



THEN DEFINE $\alpha = \min\left\{a, \ b/M\right\}$

$$\boxed{\text{UNIQUE SOL}^{\text{N}} \text{ EXISTS ON OPEN INTERVAL } (0, \alpha) \text{ WHERE } \alpha \text{ DEPENDS ON } y_0}$$

(10) LET $Ly \equiv \dfrac{d}{dt} y + p(t) y$

THEN $Ly_1 = 0$ AND $Ly_2 = g(t)$ ARE GIVEN.

BECAUSE L IS LINEAR (I.E. $L(f+g) = Lf + Lg$ AND $L(cf) = c \cdot Lf$)

WE KNOW $L(y_1 + y_2) = Ly_1 + Ly_2 = 0 + g(t) = g(t)$

AS DESIRED.